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Computational Geometric Series Model with Key Applications in Informatics

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Abstract

The current paper presents an innovative Digital Computing Model using Geometric Series and its Formulae (GSF). The generic computing model and GSF are mostly used in various scientific research fields such as information & communication technology, digital signal processing, engineering, medicine, bioinformatics, etc. particularly in computing model that performs infinitely many computational steps in finite time. In the research study, a generalized theorem, which is named as a generic digital computing-geometric series model, has been introduced and analyzed with key applications. The generalized theorem has been derived from the basic equation x = y, where x or y denotes a power of two. The digital computing-geometric series model will be very useful for researchers working on science and engineering fields for finding solutions to the real world complex problems.

Keywords: computing model, digital technology, geometric series ant its formulae, medicine, digital number systems.

Introduction
The base-2 system is a binary number system in which each binary number represents a digital number consisting of the digits 0's and/or 1's. Similarly, the base-10 system is a decimal number system in which each number designates a digital number consisting of the digits 0's, 1's, 2's, - - -, 8's, and/or 9's. In general, many number systems are in use in digital technology. The most common number systems are the binary, octal, decimal, and hexadecimal systems. The decimal and binary number systems are the most familiar to us because the decimal numbers are used in high-level computing programs (or user-level computing programs) and the binary

numbers are used in low-level programs (or machine-level programs). In digital systems the information that is being processed is usually presented in binary form.

Nowadays, the geometric series play a key role in digital number system, for example, converting a base-2 number into a base-10 number. Traditionally, geometric series have played a vital role in the early development of calculus, but today, the geometric series have many key applications in information and communication technology, signal processing, engineering, medicine, bioinformatics, etc. In the present study, a generalized theorem, which is named as a generic digital computing-geometric series model, will be introduced and analyzed with key applications.

General Form of GSF (Geometric Series and its Formulae)

Traditionally, geometric series played a key role in the early development of calculus, but today, the geometric series have many key applications in medicine, computational biology, informatics, etc.

In general, a geometric series is the sum of the terms of the geometric sequence: $a, ar, ar^2, ar^3, \cdots, ar^n, \cdots$.

Now, the sum of the geometric sequence of n terms is denoted by $S = \sum_{j=0}^{n-1} ar^j = a + ar + ar^2 + ar^3 + \cdots + ar^{n-1}$

where S denotes the sum, a the first term, r the ratio, and n the number of terms.

$$rS = ar + ar^2 + ar^3 + \cdots + ar^{n-1} + ar^n.$$

when $r \geq 1$,

$$(r-1)S = a(r^n - 1) \Rightarrow S = \frac{a(r^n - 1)}{(r-1)} \; (r \neq 1).$$

and

when $(-1 < r < 1)$ or $(|r| < 1)$,

$$(1-r)S = a(1-r^n) \Rightarrow S = \frac{a(1-r^n)}{(1-r)}, \text{ where } r \neq 1$$

$$\sum_{j=0}^{\infty} ar^j = \frac{a}{(1-r)} \; (|r| < 1)$$

$\sum_{j=0}^{n-1} r^j = 1 + r + r^2 + r^3 + \cdots + r^{n-1}$. In the geometric series, the first term shows a = 1.

Then $\sum_{j=0}^{\infty} r^j = \dfrac{1}{(1-r)}$ *when* $|r| < 1$

Geometric Series in Digital Numbers

The geometric series is very useful for converting a binary number into a decimal number.

$$\sum_i \frac{1}{2^i} \bullet \sum_j \frac{1}{2^j} = \overbrace{11111 \cdots 1111}^{n} \bullet \overbrace{1111 \cdots 11111}^{n} \quad (-n \le i \le 0 \;\&\; 1 \le j \le n)$$

$$\sum_i \frac{1}{2^i} \bullet \sum_j \frac{1}{2^j} = \cdots 11111 \bullet 11111 \cdots, \; (-\infty < i \le 0 \;\&\; 1 \le j < \infty)$$

Examples

(i) $\sum_i \frac{1}{2^i} \bullet \sum_j \frac{1}{2^j} = 100111011 \bullet 1001101101$,

$(i = -8, -5, -4, -3, -1, 0 \;\&\; j = 1, 4, 5, 7, 8, 10)$

(ii) $\sum_i \frac{1}{2^i} \bullet \sum_j \frac{1}{2^j} = 100000001 \bullet 10000001$, $(i = -8, 0 \;\&\; j = 1, 8)$

The positions of 0's need not be included for i and j

Applications of Geometric Series and its Formulae

The Use of GSF in Medicine Dosage

In this section, we discuss about the effective medicine dosage using Geometric Series and its Formulae (GSF). Let us consider a patient is given the same dose of a medicine at equally spaced time intervals. The dose concentration in the bloodstream decreases as the drug is broken down by the body. However, it does not disappear completely before the next dose is given. Let us understand the exponential decay model [1] for the concentration of a drug in a patient's bloodstream. It is assumed that the drug is administered intravenously and that the concentration of the drug in the bloodstream jumps almost immediately to its highest level, i.e. the concentration of the drug decays exponentially.

Let $Q(t)$ be a function of variable't'. Now, we use the function $Q(t)$ to represent a dose concentration at time t and Q_0 to represent the concentration just after the dose is administered intravenously. Then the exponential decay model [1] is formulated by

$$Q(t) = Q_0 \frac{1}{e^{kt}} = Q_0 e^{-kt}$$

where k is the decay constant or a property of the particular drug being used. The above model was fitted by experimental data [1].

Now, let us consider that $Q(t)$ be the first dose concentration at time t and that Q_0 the concentration at time t = 0 just after the first dose is administered intravenously. Suppose that at t = c, a second dose of the drug is given to the patient. The concentration of the drug in the bloodstream jumps almost immediately to its highest level $Q(c)$ and then the concentration is diffused so rapidly throughout the bloodstream over time (Fig.1).

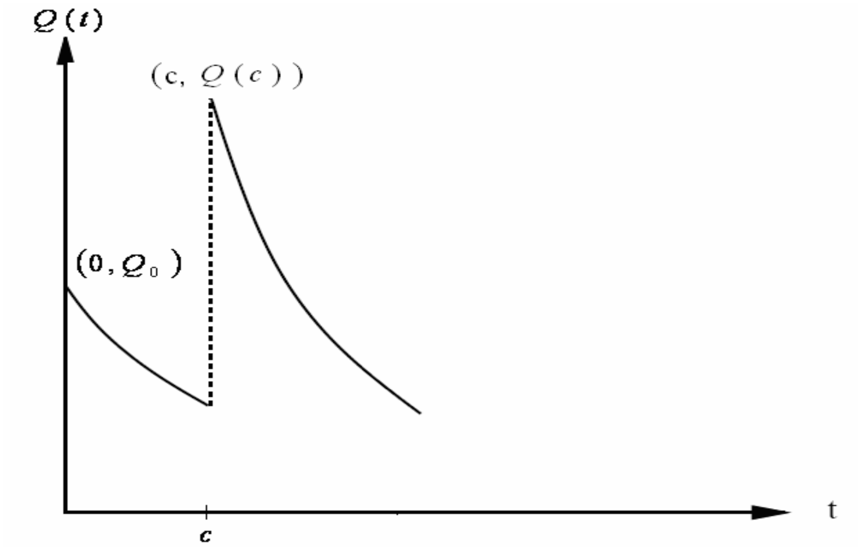

Figure 1: Dose consideration.

For example, a patient is injected a particular drug. Just after the drug is injected, the concentration is 1.5 mg/ml (milligrams per milliliter). After four hours the concentration has dropped to 0.25 mg / ml.

Here, $Q(4) = 0.25$ at t = 4 and $Q_0 = 1.5$ at t = 0

So, $Q(t) = Q_0 e^{-kt} \Rightarrow 0.25 = 1.5 e^{-4k}$

To find k, Maple commands were used [1].

Result: k = 0.4479398673.

A problem facing physicians is the fact that for most drugs, there is a concentration, m, below which the drug is ineffective and a concentration, M, above which the drug is dangerous. Thus, the concentration $Q(t)$ must satisfy the condition: $m \le Q(t) \le M$. For example, suppose that for the drug in the experiment the maximum safe concentration is 5 mg/ml, or *M*=5, and the minimum effective concentration is 0.6 mg/ml, or *m*=0.6. Then the initial dose must not produce a concentration greater than 5 mg/ml.

The expression $Q(t) = Q_0 e^{-kt}$ is valid as long as only a single dose is given. However, suppose that, at t = c, a second dose is given and that the amount of the drug administered is the same as the first dose. According to the exponential decay model, the concentration will jump immediately by an amount equal to Q_0 when the second dose is given. However, when the second dose is given, there is still some of the drug in the bloodstream remaining from the first dose. This means that to compute the concentration just after the second dose, we have to add the value Q_0 to the concentration remaining from the first dose (Fig.1). During the time between the second and third doses, the concentration decays exponentially from this value. To find the concentration after the third dose, the same process must be repeated.

At t = c, the dose concentration is calculated as $Q(c^-) = Q_0 e^{-kc}$ just before the second dose is administered intravenously.

Here, $Q(c^-) = \lim_{t \to c^-} Q(t)$.

When the second dose is administered intravenously, the concentration jumps by an increment Q_0, i.e. the concentration just after the second dose given is

$$Q(c^+) = Q_0 + Q(c^-) = Q_0 + Q_0 e^{-kc} = Q_0 \left(1 + e^{-kc}\right).$$

Note that $Q(c^-)$ denotes 'just before the new dose is administered' and $Q(c^+)$ denotes 'just after the new dose is administrated'.

The concentration then decays from this value according to the exponential decay rule [4], but with a slight twist. The twist is that the initial concentration is at t = c, instead of t = 0. One way to handle this is to write the exponential term as $e^{-k(t-c)}$ so that at t = c, the exponent is 0. If we do this, then we can write the concentration as a function of time as

$$Q(t) = Q_0 \left(1 + e^{-kc}\right) e^{-k(t-c)}$$

This function is only valid after the second dose is administered and before the third dose is given. That is, for $c \le t < 2c$.

Now, suppose that a third dose of the drug is given at t = 2c. The concentration just before the third dose is given would be $Q(2c^-)$, which is

$$Q(2c^-) = Q(c^+) e^{-kc} = Q_0 \left(1 + e^{-kc}\right) e^{-kc}$$

i.e., $Q(2c^-) = Q_0 \left(e^{-kc} + e^{-2kc}\right)$

When the third dose is given, the concentration would jump again by Q_0 and the concentration just after the third dose would be

$$Q(2c^+) = Q_0 + Q(2c^-) = Q_0 \left(1 + e^{-kc} + e^{-2kc}\right)$$

Now, suppose that a forth dose of the drug is given at t = 3c. The concentration just before the forth dose is given would be $Q(3c^-)$, which is

$$Q(3c^-) = Q(2c+) e^{-kc} = Q_0 \left(e^{-kc} + e^{-2kc} + e^{-3kc}\right)$$

When the third dose is given, the concentration would jump again by Q_0 and the concentration just after the third dose would be

$$Q(3c^+) = Q_0 + Q(3c^-) = Q_0\left(1 + e^{-kc} + e^{-2kc} + e^{-3kc}\right)$$

Let us consider the process is continued up to n-th dose,

i.e. $\overbrace{0, 1, 2, 3, \cdots\ n-1}^{n-doses}$

The concentration just before the n-th dose of the drug would be

$$Q((n-1)c^-) = Q_0 \sum_{j=1}^{n-1} e^{-jkc} \qquad (1)$$

The concentration just after the n-th dose of the drug would be

$$Q((n-1)c^+) = Q_0 \sum_{j=0}^{n-1} e^{-jkc} \qquad (2)$$

Let $r = e^{-kc}$ (3)

Note that $0 < r < 1$, since k and c are both positive constants.

From the geometric series (1) and (2) , we formulate as

$$Q((n-1)c^-) = Q_0 \sum_{j=1}^{n-1} e^{-jkc} = Q_0\left(\frac{r - r^n}{1 - r}\right) \qquad (4)$$

and

$$Q((n-1)c^+) = Q_0 + Q((n-1)c^-) = Q_0 \sum_{j=0}^{n-1} e^{-jkc} = Q_0\left(\frac{1 - r^n}{1 - r}\right) \qquad (5)$$

The equations (4) and (5) are formulae for the partial sum of a geometric series.

Suppose a treatment for a patient is continued indefinitely. Then the equation (5) becomes

$$Q((n-1)c^+) = \lim_{n\to\infty} Q_0\left(\frac{1 - r^n}{1 - r}\right) = Q_0\left(\frac{1}{1 - r}\right) \quad (\because\ 0 < r < 1).$$

Now, we conclude from the results that the minimum concentration is the concentration just before the second dose is given, i.e. $Q_{\min} = Q_0 r$

and that the maximum concentration is the concentration just after the last dose is given, i.e.

$$Q_{\max} \leq \left(\frac{Q_0}{1 - r} \right)$$

Geometric Series in Digital Signal Processing (DSP)

In digital signal processing [5], the ratio r in the geometric series is often a complex exponential variable of the form $e^{i\theta}$, where i = $\sqrt{-1}$. On the complex plane (Fig.2), $e^{i\theta}$ becomes a unit vector at angle θ measured counterclockwise from the positive real axis. Let N be the number of partitions in the complex plane. Then $\theta = \frac{2\pi}{N}$ and $e^{i\theta} = e^{i\frac{2\pi}{N}}$, i.e. the ratio $r = e^{i\frac{2\pi}{N}}$ and the first element a = 1 (refer to the section 1.1 General Form of GSF (Geometric Series and Formulae).

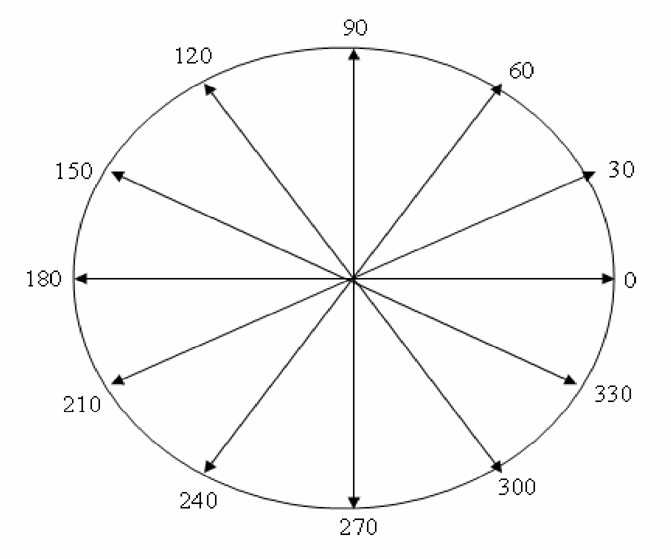

Figure 2: Complex plane.

Now, $$\sum_{n=0}^{N-1} e^{i\frac{2\pi n}{N}} = \frac{1 - e^{i2\pi}}{1 - e^{i\frac{2\pi}{N}}} = 0 \quad , (\because e^{2\pi} = e^0 = 1) \qquad (6)$$

This expression in digital signal processing is simplified using the geometric series.

Digital Computing-Geometric Series Model

Definition: Digital Computing-Geometric Series (DCGS)

Digital computing-geometric series is defined as a geometric series composed of finite or infinite number of elements (or terms). Here, each term (or element) designates a multiple of 2 or $\frac{1}{2}$ and the number 2 is the base to the digital numbers consisting of the digits 0's and 1's.

Generalized Theorem

$\sum_{j=i}^{n} \frac{1}{2^j} = x - \frac{1}{2^n}$ is derived from the equation x = y, where $x = \frac{1}{2^k}$, i = k + 1 (k ∈ Z, set of integers).

Proof

Given that k ∈ Z, i.e., k = - - -, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, - - - .

Here, x = - - -, $\frac{1}{2^{-4}}, \frac{1}{2^{-3}}, \frac{1}{2^{-2}}, \frac{1}{2^{-1}}, \frac{1}{2^{0}}, \frac{1}{2^{1}}, \frac{1}{2^{2}}, \frac{1}{2^{3}}, \frac{1}{2^{4}}$, - - -,

i.e., x = - - -, 16, 8, 4, 2, 1, $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$, - - -

The following table (refer to Table 1.) and Lemma 1, Lemma 2, Lemma 3, and Lemma 4, depict with systematic way or algorithmic approach for proving the generalized theorem, which is designates as a generic digital computing-geometric series model (or simply, digital computing-geometric series model).

Table 1: Algorithmic approach for proving the above theorem.

x = y = $\frac{1}{2^k}$	k	x – $\frac{1}{2^n}$	k + 1 = i	j = i
- - -	- - -	- - -	- - -	- - -
16 = 16 = $\frac{1}{2^{-4}}$	-4	16 - $\frac{1}{2^n}$	-4 + 1 = -3	-3
8 = 8 = $\frac{1}{2^{-3}}$	-3	8 - $\frac{1}{2^n}$	-3 + 1 = -2	-2
4 = 4 = $\frac{1}{2^{-2}}$	-2	4 - $\frac{1}{2^n}$	-2 + 1 = -1	-1
2 = 2 = $\frac{1}{2^{-1}}$	-1	2 - $\frac{1}{2^n}$	-1 + 1 = 0	0
1 = 1 = $\frac{1}{2^{0}}$	0	1 - $\frac{1}{2^n}$	0 + 1 = 1	1
$\frac{1}{2} = \frac{1}{2} = \frac{1}{2^{1}}$	1	$\frac{1}{2} - \frac{1}{2^n}$	1 + 1 = 2	2

$\frac{1}{4}=\frac{1}{4}=\frac{1}{2^2}$	2	$\frac{1}{4}-\frac{1}{2^n}$	2 + 1 = 3	3
$\frac{1}{8}=\frac{1}{8}=\frac{1}{2^3}$	3	$\frac{1}{8}-\frac{1}{2^n}$	3 + 1 = 4	4
$\frac{1}{16}=\frac{1}{16}=\frac{1}{2^4}$	4	$\frac{1}{16}-\frac{1}{2^n}$	4 +1 = 5	5
- - -	- - -	- - -	- - -	- - -

Lemma 1

$\sum_{j=0}^{n} \frac{1}{2^j} = 2 - \frac{1}{2^n}$ is derived from the equation 2 = 2.

Proof

x = y $\Rightarrow$ 2 = 2

2 = 2 $\Rightarrow$ 2 = 1 + 1

$$\Rightarrow \quad 2 = 1 + \frac{1}{2} + \frac{1}{2}$$

$$\Rightarrow \quad 2 = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} \qquad (7)$$

The expression (7) with finite number of terms can be expanded further, i.e.,

$$\Rightarrow \quad 2 = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{64} \qquad (8)$$

Using expression (8), the following geometric series can be formulated:

$$2 = 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots\cdots + \frac{1}{2^n} + \frac{1}{2^n}$$

$$\Rightarrow \quad 2 - \frac{1}{2^n} = 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots\cdots + \frac{1}{2^n}$$

$$x = y \Rightarrow \sum_{j=0}^{n} \frac{1}{2^j} = 2 - \frac{1}{2^n}$$

$$\therefore \quad \sum_{j=0}^{n} \frac{1}{2^j} = 2 - \frac{1}{2^n} \qquad (9)$$

Similarly, we can get the result (9) using generalized theorem:

Here, x = y =2, i.e. $x = \frac{1}{2^{-1}}$

Then k = -1, and i = k + 1 = 0 $\left(\because \quad x = \frac{1}{2^k} = \frac{1}{2^{-1}} \right)$

By substituting the values of x, k, and i in the equation

$\sum_{j=i}^{n} \frac{1}{2^j} = \frac{1}{2^k} - \frac{1}{2^n}$, we can get

$$\sum_{j=0}^{n} \frac{1}{2^{j}} = \frac{1}{2^{-1}} - \frac{1}{2^{n}} = 2 - \frac{1}{2^{n}}.$$

Lemma 2

$\sum_{j=1}^{n} \frac{1}{2^{j}} = 1 - \frac{1}{2^{n}}$ is derived from the equation 1 = 1.

Proof

It is given that x = y =1, i.e. $x = \frac{1}{2^{0}}$

Then k = 0 and i = k + 1 = 1 $\left(\because \quad x = \frac{1}{2^{k}} = \frac{1}{2^{0}} \right)$

Now, x = y ⇒ 1 = 1

$$\Rightarrow \quad 1 = \frac{1}{2} + \frac{1}{2}$$

$$\Rightarrow \quad 1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{4} \qquad (10)$$

The expression (10) with finite number of elements can be expanded further, i.e.

$$\Rightarrow \quad 1 = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \frac{1}{64} + \frac{1}{64} \qquad (11)$$

Using expression (11), the following geometric series is formed:

$$1 = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \frac{1}{2^4} + \cdots\cdots + \frac{1}{2^n} + \frac{1}{2^n}$$

$$\Rightarrow \; 1 - \frac{1}{2^n} = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots\cdots + \frac{1}{2^n}$$

$$x = y \Rightarrow \sum_{j=1}^{n} \frac{1}{2^{j}} = 1 - \frac{1}{2^{n}}$$

$$i.e., 1 = 1 \Rightarrow \sum_{j=1}^{n} \frac{1}{2^{j}} = 1 - \frac{1}{2^{n}} \qquad (12)$$

Similarly, we can have the result (12) using generalized theorem:

Here, x = y = 1, i.e. $x = \frac{1}{2^{0}}$

Then k = 0, and i = k + 1 = 1 $\left(\because \quad x = \frac{1}{2^{k}} = \frac{1}{2^{0}} \right)$

By substituting the values of x, k, and i in the generic equation

$\sum_{j=i}^{n} \frac{1}{2^{j}} = \frac{1}{2^{k}} - \frac{1}{2^{n}}$, we can get

$$\sum_{j=1}^{n} \frac{1}{2^{j}} = \frac{1}{2^{0}} - \frac{1}{2^{n}} = 1 - \frac{1}{2^{n}}.$$

Lemma 3

$\sum_{j=-n}^{0} \frac{1}{2^{j}} = 2^{n+1} - 1$ is derived from the equation $2^{n+1} = 2^{n+1}$.

Proof

$x = y \Rightarrow 2^{n+1} = 2^{n+1}$

$\Rightarrow 2^{n+1} = 2^{n} + 2^{n-1} + 2^{n-2} + \cdots + 2^{2} + 2^{1} + 2^{0} + 2^{0}$

$\Rightarrow 2^{n+1} - 1 = 2^{n} + 2^{n-1} + 2^{n-2} + \cdots + 2^{2} + 2^{1} + 1$

$$\therefore \sum_{j=-n}^{0} \frac{1}{2^{j}} = 2^{n+1} - 1 \qquad (13)$$

Similarly, we can have the result (13) using generalized theorem:

Here, $x = y = 2^{n+1}$, i.e. $x = \frac{1}{2^{k}} = \frac{1}{2^{-(n+1)}}$

Then k = -(n+1), and i = k + 1 = -n -1 + 1 = -n.

By substituting the values of x, k, and i in the equation

$\sum_{j=i}^{n} \frac{1}{2^{j}} = \frac{1}{2^{k}} - \frac{1}{2^{n}}$, we can have

$$\sum_{j=-n}^{0} \frac{1}{2^{j}} = \frac{1}{2^{-(n+1)}} - \frac{1}{2^{0}} = 2^{n+1} - 1 \qquad (14)$$

Lemma 4

$\sum_{j=n+1}^{2n} \frac{1}{2^{j}} = \frac{1}{2^{n}} - \frac{1}{2^{2n}}$ is derived from the equation $\frac{1}{2^{n}} = \frac{1}{2^{n}}$

Proof

$x = y \Rightarrow \frac{1}{2^{n}} = \frac{1}{2^{n}}$

$\Rightarrow \frac{1}{2^{n}} = \frac{1}{2^{n+1}} + \frac{1}{2^{n+2}} + \cdots + \frac{1}{2^{n+(n-1)}} + \frac{1}{2^{2n}} + \frac{1}{2^{2n}}$

$\Rightarrow \frac{1}{2^{n}} - \frac{1}{2^{2n}} = \frac{1}{2^{n+1}} + \frac{1}{2^{n+2}} + \cdots + \frac{1}{2^{n+(n-1)}} + \frac{1}{2^{2n}}$

$$\therefore \sum_{j=n+1}^{2n} \frac{1}{2^{j}} = \frac{1}{2^{n}} - \frac{1}{2^{2n}} \qquad (15)$$

Similarly, we can have the result (15) using generalized theorem:

Here, $x = y = \frac{1}{2^{n}}$, i.e. $x = \frac{1}{2^{k}} = \frac{1}{2^{n}}$

Then k = n, and i = k + 1 = n+1

By substituting the values of x, k, and i in the equation

$\sum_{j=i}^{n} \frac{1}{2^{j}} = \frac{1}{2^{k}} - \frac{1}{2^{n}}$, we can have

$$\sum_{j=n+1}^{2n} \frac{1}{2^{j}} = \frac{1}{2^{n}} - \frac{1}{2^{2n}}$$

From Table 1 and by Lemma 1, Lemma2, Lemma 3, and Lemma 4, we conclude that the generalized theorem $\sum_{j=i}^{n} \frac{1}{2^j} = x - \frac{1}{2^n}$ is derived from the equation x = y, where $x = \frac{1}{2^k}$, i = k + 1 (k ∈ Z, set of integers).

Infinite Geometric Series Model

Now, it is understood that $\sum_{j=i}^{n} \frac{1}{2^j} = x - \frac{1}{2^n}$ is derived from the equation x = y, where $x = \frac{1}{2^k}$, i = k + 1 (k ∈ Z, set of integers).

Next, generic infinite geometric series is formed:

In the generalized theorem, x is a constant for each geometric series.

Let x =c.

Then, $\sum_{j=i}^{n} \frac{1}{2^j} = c - \frac{1}{2^n}$ for each i,

where i = - - - -, -3, -2, -1, 0, 1, 2, 3, 4, - - - -.

$$\sum_{j=i}^{\infty} \frac{1}{2^j} = \lim_{n \to \infty} \sum_{j=i}^{n} \frac{1}{2^j}$$

$$= \lim_{n \to \infty} \left(c - \frac{1}{2^n} \right)$$

$$= \lim_{n \to \infty} c - \lim_{n \to \infty} \frac{1}{2^n} \quad \left(\because \lim_{n \to \infty} \frac{1}{n} = 0 \right)$$

= c – 0

Thus, $\sum_{j=i}^{\infty} \frac{1}{2^j} = c$

where c = - - -, 16, 8, 4, 2, 1, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, - - - ,

Example

Let x = y =2, i.e. $x = \frac{1}{2^{-1}}$

Then k = -1, and i = k + 1 = 0 $\left(\because x = \frac{1}{2^k} = \frac{1}{2^{-1}} \right)$

Then $\sum_{j=0}^{\infty} \frac{1}{2^j} = \lim_{n \to \infty} \sum_{j=0}^{n} \frac{1}{2^j}$

$$= \lim_{n \to \infty} \left(2 - \frac{1}{2^n} \right) \quad (\because c = x = 2)$$

$$= \lim_{n \to \infty} 2 - \lim_{n \to \infty} \frac{1}{2^n}$$

= 2 – 0

$$\sum_{j=0}^{\infty} \frac{1}{2^j} = 2 \qquad (16)$$

Digital Number Systems with Geometric Series

The geometric series play a key role in digital number systems in converting a base-2 number into a base-10 number.

Non-Fractional Numbers

$\overset{n}{1}\;\overset{n-1}{1}\;\overset{n-2}{1}\;\cdots\;\overset{2}{1}\;\overset{1}{1}\;\overset{0}{1}$ is a binary number and its positions of the digits are n, n-1, n-2, - - -, 2, 1, 0. The geometric series corresponding to the binary number 111- - - 111 is

$$2^{n} + 2^{n-1} + 2^{n-2} + \cdots + 2^{2} + 2^{1} + 2^{0}$$

(OR)

$$\frac{1}{2^{-n}} + \frac{1}{2^{-(n-1)}} + \frac{1}{2^{-(n-2)}} + \cdots + \frac{1}{2^{-2}} + \frac{1}{2^{-1}} + \frac{1}{2^{0}} = \sum_{j=-n}^{0} \frac{1}{2^{j}}$$

i.e.,

$$\sum_{j=-n}^{0} \frac{1}{2^{j}} = 1 + 2 + 4 + 8 + 16 + \cdots + 2^{(n-2)} + 2^{(n-1)} + 2^{n}$$

From the equation (14), we get

$$\sum_{j=-n}^{0} \frac{1}{2^{j}} = 2^{n+1} - 1$$

Thus, the decimal number corresponding to the binary number

$$\overset{n}{1}\;\overset{n-1}{1}\;\overset{n-2}{1}\;\cdots\;\overset{2}{1}\;\overset{1}{1}\;\overset{0}{1} \text{ is } 2^{n+1} - 1 \qquad (17)$$

Fractional Number

$0 \bullet \overset{1}{1}\;\overset{2}{1}\;\overset{3}{1}\cdots\overset{n-2}{1}\;\overset{n-1}{1}\;\overset{n}{1}$ is a fractional binary number and its positions of the digits (except the digit 0 before the dot in the fractional binary number) are 1, 2, 3, - - -, n-2, n-1, n. The geometric series corresponding to the fractional binary number 0.111- - - 111 is

$$0 \bullet \frac{1}{2^{1}} + \frac{1}{2^{2}} + \frac{1}{2^{3}} + \cdots + \frac{1}{2^{n-2}} + \frac{1}{2^{n-1}} + \frac{1}{2^{n}} = 0 \bullet \sum_{j=1}^{n} \frac{1}{2^{j}}$$

$$0 \bullet \sum_{j=1}^{n} \frac{1}{2^{j}} = 0 \bullet 1 - \frac{1}{2^{n}}$$

Thus, the fractional decimal number corresponding to the fractional binary

$$\text{number } 0 \bullet \overbrace{111 \;\cdots\; 111}^{n} \text{ is } 0 \bullet 1 - \frac{1}{2^{n}} \qquad (18)$$

By summing the equations (17) and (18), we see that the decimal number corresponding to the binary number

$$\overset{n}{1}\;\overset{n-1}{1}\;\overset{n-2}{1}\;\cdots\;\overset{2}{1}\;\overset{1}{1}\;\overset{0}{1}\bullet\overset{1}{1}\;\overset{2}{1}\;\overset{3}{1}\cdots\overset{n-1}{1}\;\overset{n}{1} \; is \; 2^{n+1} - 1 \bullet 1 - \frac{1}{2^{n}}.$$

Performance of Computing Machine

We know that a computing machine performs infinitely many computational steps in finite time. Now, let us imagine that the computing machine performs first computation step in (say) 1 minute, the second step in ½ minute, the third step in ¼ minute, and so on. By summing the infinite computation steps, we get an infinite geometric series such as 1 + ½ + ¼ + - - -.

$$\sum_{j=0}^{\infty} \frac{1}{2^j} = 1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots$$

From the equation (16), we get $\sum_{j=0}^{\infty} \frac{1}{2^j} = 2$.

Thus, by summing the infinite computation steps, we see that the machine performs infinitely many computation steps in a total of 2 minutes.

Bacterium's Partition Process

A geometric sequence is 1, 2, 4, 8, 16, 32, etc. in which each number is obtained by doubling the previous one. As to biological relevance [2], imagine a single bacterium that divides into two. These two divides into four, these four into eight, and so on. After n divisions, there are 2^n bacteria if all survive.

The time taken for a bacterium's 'n' divisions is computed in the following manner. Let us assume that the first division takes place in (say) 1 second, the second division in ½ second, the third division in ¼ second, - - - , n^{th} division in $\frac{1}{2^n}$ second. The total time taken for n divisions is denoted by

$$1 + \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + \cdots + \frac{1}{2^n} = \sum_{j=0}^{n} \frac{1}{2^j} = 2 - \frac{1}{2^n} \text{ sec}.$$

Conclusion

In the research study, an innovative Digital Computing Model using GSF has been developed and important applications of GSF such as effective medicine dosage, digital signal processing, binary-decimal conversion, performance of computing machine, and bacterium's partition process have been provided in detail. This digital computing model will be very useful for scientists and researchers for finding solutions to various complex problems. In the present paper, a generalized theorem, which is named as a generic digital computing-geometric series model, has been introduced and proved, i.e. $\sum_{j=i}^{n} \frac{1}{2^j} = x - \frac{1}{2^n}$ has been derived from the equation x = y, where $x = \frac{1}{2^k}$, i = k + 1 (k ∈ Z, set of integers).

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